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**ENTRY AND R&D COSTS  
IN COMPETITIVE PROCUREMENTS AND CONTRACTING**

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ABSTRACT

A model of competitive procurements and contracting is presented. The key features of the model include pre-contract R&D, an endogenous number of symmetric firms, and a first-price sealed-bid procurement auction. The unique symmetric perfect free-entry equilibrium is characterized. If the R&D technology is variable scale with constant marginal returns, it is socially optimal for one firm to do all of the R&D and production. However, since the buyer considers only his own cost of procurement, the buyer will prefer to allow free entry, and the number of firms will usually be larger than is socially optimal. If the R&D technology is fixed-scale, the buyer's choices will be socially optimal if the buyer's opportunity cost of an alternative procurement is high. On the other hand, if the opportunity cost is low the buyer will choose a reservation price lower than the socially optimal value and a number of firms no larger than the socially optimal number. Certainly, the type of R&D technology plays an important role in determining optimal R&D and procurement policies for the buyer and for society.

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# ENTRY AND R&D COSTS

## In Competitive Procurements and Contracting

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### 1. Introduction

In competitive procurements and contracting, potential firms usually participate in R&D activities. Some firms may not find it profitable to enter into competitive bidding because of the high costs of precontract R&D. In many procurement cases, only a small number of potential firms choose to submit bids (see Besen and Terasawa 1987). According to Hendricks, Porter, and Boudreau (1987), potential firms decide how much information to collect before participating in competitive bidding. In some instances, a firm may decide not to participate because the cost of searching for the information necessary to submit a bid exceeds the expected gain. In general, the decisions to acquire information and to submit bids depend on the R&D process, the costs of R&D, the costs of preparing bids, and the type of competitive bidding procedure in place.

The existing literature on auctions and procurements,<sup>1</sup> except for French and McCormick (1984), and McAfee and McMillan (1987a), typically assumes that the number of bidders is exogenous and constant.<sup>2</sup> For a given number of firms, a buyer with incomplete knowledge about the firms' production costs should procure the goods at the level at which the marginal benefit equals the marginal virtual cost.<sup>3</sup> The buyer discriminates as a monoposonist. Asymmetry of information causes a welfare loss for the buyer. The more the firms compete for the procurement contract, the less the welfare loss. When the number of firms goes to infinity, the welfare loss disappears; hence the most efficient outcome is reached. In this literature, entry behavior in auction and procurement processes has not been examined carefully. Although French and McCormick (1984), and McAfee and McMillan (1987a) have considered precontract costs (or fixed entry costs) and entry equilibria, prebidding R&D decisions have not been formally modelled.<sup>4</sup> On the one hand, if fewer firms participate in the competitive bidding, the contract will be more profitable to the winning firm and each firm will tend to invest more in R&D. If the expected profit of the winning firm is positive,

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more firms will enter the auction. On the other hand, the buyer may want to control the firms' R&D decisions through the choice of the contract auction rules. What is the equilibrium number of bidders under free entry and how does each potential firm make precontract R&D decisions? Is free entry of firms an optimal policy for the buyer? Moreover, is it socially optimal? These are the questions that I intend to answer in this paper.

A model of competitive procurement with precontract R&D is considered in this paper. The number of firms is viewed as an endogenous variable in the model. I distinguish active firms (or informed firms) from actual bidders. A firm is active if it invests in R&D and becomes informed about demand and production cost. An actual bidder is a firm that submits a bid in the auction for the production contract. Similar to Tan (1989), the R&D activity by each firm is formally modelled as a stochastic process with certain R&D costs. I also allow for each firm a bid-preparation cost similar to that in Samuelson (1985). These R&D costs and bid-preparation costs affect the number of informed firms and the number of actual bidders. Under free entry, the equilibrium number of informed firms, the expected number of actual bidders, and the level of investment in R&D, are *simultaneously* determined and depend on the R&D costs, bid-preparation costs, and the type of auction.

The next section describes the model and the equilibrium concept I am going to use. Then I show the existence and uniqueness of symmetric perfect free-entry equilibrium under the first-price sealed-bid auction with a given reservation price. Without bid-preparation costs, the total equilibrium expenditure on R&D among all firms decreases with the marginal cost of R&D. When the marginal cost of R&D approaches zero, the total expenditure by all firms goes to infinity no matter how large the fixed cost of R&D. On the other hand, when the marginal cost of R&D is relatively high, it will be very costly for any firm to conduct any R&D activity. Thus, the marginal cost of R&D is the key determinant of the total R&D expenditure. Without a fixed R&D cost, free entry causes an infinite number of firms to enter the R&D process. But with a positive fixed R&D cost, only a finite number of firms will decide to invest in R&D. The higher the fixed cost of R&D, the fewer the equilibrium number of active firms. In other words, the fixed cost of R&D plays an important role in determining the equilibrium number of active firms and the expected number of actual bidders.

From the point of view of the buyer's optimal strategy, I show the following: First, consider the fixed-scale R&D technology. If the buyer's opportunity cost of procuring the good somewhere else is relatively high, no reservation price is necessary for the buyer. If the buyer's opportunity cost is relatively low, however, he should choose a reservation price which is lower than the opportunity cost. This optimal reservation price is higher than the reservation price the buyer would have chosen if there were no fixed R&D cost. Although there exists a distortion of the efficient outcome, the presence of positive fixed costs of R&D makes that distortion smaller. In both cases, the optimal number of informed firms for the buyer enter the procurement process under free entry. That is, free entry is optimal for the buyer conditional on the appropriate choice of a reservation price. Second, when R&D is subject to constant marginal returns to scale on expenditure, if the buyer is able to control each firm's R&D investment costlessly, he does not want to leave the R&D decisions and entry decisions to the firms. In general, he wants each firm to invest more in R&D than it wants to. Thus, when R&D decisions are not observable by the buyer, a moral hazard problem arises. Taking

each firm's R&D decision as a constraint, the best reservation price for the buyer may be either higher than or lower than his opportunity cost of an alternative procurement.

Social optimality is characterized by the minimization of the total expected social costs. Under either the variable scale R&D technology with constant marginal returns or the fixed-scale R&D technology, social optimality requires the buyer to set his reservation price equal to the minimum of his opportunity cost and the highest possible production cost observation among all firms. Also, in contrast to the buyer's preferences, when each firm's R&D decision is subject to constant marginal returns to scale on expenditure, society prefers only one firm to conduct all of the R&D and production. The comparison between the buyer's optimum and the social optimum shows that the R&D technology plays an important role in determining the optimal R&D and procurement policies for the buyer and for society.

## 2. The Model

There is a single buyer (e.g. the government) who seeks to procure one unit of a certain novel good or service. The buyer wants to minimize the expected total costs of this procurement.

There are many potential firms; each of which can produce a unit of the good at a potential unknown cost  $y$ . Each firm can invest in R&D for information about cost reduction and will observe a potential cost  $y$  which is drawn from a same random distribution

$$H(y | x) = 1 - [1 - F(y)]^x \quad (1a)$$

with the fixed support  $[\underline{y}, \bar{y}]$ ,  $\bar{y} > \underline{y} \geq 0$ , where  $F(y)$  is a continuously differentiable cumulative distribution function with support  $[\underline{y}, \bar{y}]$  and density function  $f(y)$ , and  $x \in [0, +\infty)$  is the level of investment in R&D.<sup>5</sup> I assume  $f(y) > 0$  for all  $y \in [\underline{y}, \bar{y}]$  and  $y + F(y) / f(y)$  is increasing in  $y$ . Let  $G(y) \equiv 1 - F(y)$  for all  $y$ . The R&D cost is assumed to be linear and the same for all firms:

$$C(x) = C_1 x + C_2, \quad (1b)$$

where  $C_1 \geq 0$  is the marginal cost of R&D investment and  $C_2 \geq 0$  is the fixed cost of R&D. This R&D activity can be viewed as an independent experimental drawing process. For example, if the firm invests one unit (or one experiment)  $x = 1$ , a cost level  $y$  will be observed from the distribution  $F(y)$  at the cost  $C_1 + C_2$ . If the firm repeats this experiment  $x = k$  times, each additional experiment costs  $C_1$ . Then  $k$  numbers of production cost  $(y_1, \dots, y_k)$  will be observed at the cost  $C_1 k + C_2$ . The minimum cost level  $y$  of  $(y_1, \dots, y_k)$  is subject to the distribution of the lowest-order statistic  $H(y | k)$ , which has the form of (1a).

In this paper competitive procurement is modelled as a three-stage process. In the first stage, the buyer announces and commits to the general rules of procurements. I consider the first-price sealed-bid auction with an announced reservation price.<sup>6</sup> In the second stage R&D is conducted during which each firm invests in R&D and acquires information about the production cost. In the final stage, a competitive bidding procedure is conducted in which the buyer procures the good via the sealed-bid auction announced at the beginning. More specifically, this process can be described

as follows. First, the buyer announces the rules of the sealed-bid auction including a reservation price  $r$  which is no higher than the highest possible cost level  $\bar{y}$ . The lowest bid will be accepted unless it is below  $r$ . Second, the firm will calculate its expected profit from bidding and decide to invest in R&D if this profit is no less than its R&D cost. Third, based on the observed production cost information  $y$ , the firm will bid unless the expected profit is less than the bid preparation cost  $K$  that each has to pay to participate in the bidding process. The winner is then chosen as the contractor for production. The buyer is able to procure the good elsewhere at the cost  $y_0$  if the lowest bid is higher than the reservation price  $r$ . A special case is when  $y_0$  is very high, which means that there are no substitutes available for the buyer. Let  $y_m = \min(\bar{y}, y_0)$  represent the minimum of the highest possible production cost  $\bar{y}$  which the firms observe and the buyer's opportunity cost  $y_0$ .

Suppose  $x$  is predetermined at a fixed sunk cost  $C > 0$  of R&D and there is no bid preparation cost,  $K = 0$ . Each firm will make a take-it-or-leave-it decision whether to try to reduce its production cost. Each firm will observe a production cost  $y$  and believes that other firms' observations of production cost are drawn independently from a cumulative distribution  $F(y)$  with the support  $[y, \bar{y}]$ . This is the case in French and McCormick (1984) and McAfee and McMillan (1987). On the other hand, suppose that each firm learns its production cost without any R&D cost. Given a fixed number of firms and their types, each firm submits a bid upon paying a cost  $K > 0$ . This is the case in Samuelson (1985). In order to see the importance of the type of R&D technology, in this paper, I consider both the fixed-scale R&D technology and the variable scale R&D technology subject to constant marginal returns to scale on expenditure as described in (1a) and (1b). Therefore, the present study can also be viewed as a generalization of the models by French and McCormick (1984), McAfee and McMillan (1987), and Samuelson (1987).

In the next section, I will analyze a free-entry equilibrium for a particular reservation price of the buyer in a sealed-bid auction. A perfect free-entry equilibrium of the final two-stage game consists of a market structure  $n$ , an investment strategy  $(x_1, \dots, x_n)$ , and a bidding strategy  $(B_1(y_1), \dots, B_n(y_n))$  such that the following apply: (i) the bidding strategy  $(B_1(y_1), \dots, B_n(y_n))$  is a Bayes-Nash equilibrium, (ii) the investment strategy  $(x_1, \dots, x_n)$  is a noncooperative Nash equilibrium, taking into account the optimal bidding strategy, (iii) each of  $n$  firms in the market must anticipate nonnegative profits, and (iv)  $n + 1$  firms would earn negative profits. Entry decisions are simultaneous, not sequential. Under the fixed-scale R&D technology, it is easy to see that equilibria in both (i) and (ii) are symmetric. Under the variable, constant marginal return R&D technology, however, asymmetric equilibria may exist.<sup>7</sup> I will only consider symmetric equilibria in both (i) and (ii) and call  $(n, x, B(y))$  a symmetric perfect free-entry equilibrium. I will show that, for a given reservation price, there exists a unique symmetric perfect free-entry equilibrium. In Section 4, we will see that the buyer prefers the free entry of firms and I will calculate which symmetric perfect free-entry equilibrium he should select by choosing an appropriate reservation price. Considerations from the point of view of social optimality are discussed in Section 5.

### 3. Symmetric Perfect Free-Entry Equilibrium

Given the rules of the sealed-bid auction with a reservation price  $r$ , suppose that a firm  $i$  believes that  $n$  firms including itself might invest in R&D and compete in the contract auction. I will show how the equilibrium number of firms is determined later on. Since each firm is assumed to have the same R&D technology (1a) and (1b), firm  $i$  invests  $x_i$  in R&D at a cost of  $C(x_i)$  and privately learns the new production cost  $y_i$  of supplying the good, which is independently drawn from the distribution  $H(y_i | x_i)$ . Consider symmetric noncooperative Nash equilibria  $x_i = x_j = x$  for all  $i$  and  $j$ . Firm costs are generated independently from a common distribution function  $H(y | x)$  with the support  $[\underline{y}, \bar{y}]$ .

Suppose that in the auction firm  $i$  uses a strategy  $B_i = B_i(y_i)$ ,<sup>8</sup> which is strictly increasing in  $y_i$ ,  $i = 1, \dots, n$ . Firm  $i$  with cost observation  $y_i$  will generate the following profit from bidding by submitting  $B_i$ :

$$\begin{aligned}\pi_i(B_i, y_i) &= (B_i - y_i) \text{Prob}(\text{winning}) \\ &= (B_i - y_i) \left[ 1 - H(B_i^{-1}(B_i) | x) \right]^{n-1}.\end{aligned}$$

Consider symmetric bidding strategies  $B_i(y) = B_j(y) = B(y)$  for any  $i$  and  $j$  as the Bayes-Nash equilibrium. By the Envelope Theorem, at the Bayes-Nash equilibrium

$$\frac{d\pi(B(y), y)}{dy} = -[1 - H(y | x)]^{n-1}. \quad (2)$$

The submission of a bid requires the expenditure of  $K$  in preparation costs. Free-exit implies  $\pi(B(y), y) - K \geq 0$ . Thus, the firm will not bid if costs are above some break-even level  $\hat{y}$ . The marginal firm  $\hat{y}$  is indifferent between entering a bid or not. If the marginal firm makes a bid, the optimal bid is the reservation price  $B(\hat{y}) = r$ . The probability that the marginal firm  $\hat{y}$  wins is  $[1 - H(\hat{y} | x)]^{n-1} = [1 - F(\hat{y})]^{(n-1)x} \equiv G(\hat{y})^{(n-1)x}$  and the marginal firm's expected profit will be  $\pi(B(\hat{y}), \hat{y}) = (r - \hat{y})G(\hat{y})^{(n-1)x}$ . Thus, the marginal firm  $\hat{y}$  is determined by the following free-exit condition (FE):

$$(r - \hat{y})G(\hat{y})^{(n-1)x} = K. \quad (\text{FE})$$

Then, from (2) and (FE), we have

$$\pi(B(y), y) = K + \int_{\hat{y}}^y G(t)^{(n-1)x} dt$$

for all  $y \leq \hat{y}$ . The firm with cost  $y > \hat{y}$  will not bid because its expected profit from bidding will be less than the bid preparation cost  $K$ . From (FE),  $\hat{y}$  is strictly lower than the reservation price  $r$  because of the positive bid preparation cost. If  $K = 0$  then  $\hat{y} = r$ . That is, if no such cost existed, the marginal firm would be the firm with a cost observation equal to the reservation price.

On the other hand, we know that, at the symmetric bidding equilibrium  $B = B(y)$ ,

$$\pi(B(y), y) = (B(y) - y)G(y)^{(n-1)x}.$$

Comparing the above two expressions, we can easily write the equilibrium bidding function  $B(y)$  as the following:

$$B(y) = y + \frac{K}{G(y)^{(n-1)x}} + \frac{\int_y^{\hat{y}} G(t)^{(n-1)x} dt}{G(y)^{(n-1)x}} \quad (3)$$

for all  $y \in [y, \hat{y})$  and  $B(\hat{y}) = r$ . Because of the bid-preparation cost  $K$  and the firms' private information about  $y$ , each firm intends to bid a higher level than the true production cost  $y$ . The equilibrium bid function consists of the true production cost  $y$ , the information cost, and the bid preparation cost. From (3), the equilibrium bidding function  $B(y)$  is completely determined by  $n, x$ , and  $\hat{y}$ .

Suppose firm  $i$  invests  $x_i$  in R&D. Since  $(n-1)x$  in the expression of  $\pi(B(y), y)$  is the total expenditure on R&D by the other  $n-1$  firm and independent of  $x_i$ , firm  $i$ 's total expected profits from bidding will be

$$\int_y^{\hat{y}} [\pi(B(y), y) - K] dH(y | x_i) = \int_y^{\hat{y}} G(t)^{(n-1)x} H(t | x_i) dt$$

given that each other firm chooses  $x$ . At the symmetric Nash equilibrium, firm  $i$  will choose  $x_i = x$  such that the marginal expected profit equals the marginal cost of investment in R&D. Formally, we have

$$\int_y^{\hat{y}} G(t)^{nx} \ln G(t) dt + C_1 = 0. \quad (\text{R\&D})$$

The second order condition is satisfied because  $-\int_y^{\hat{y}} G(t)^{(n-1)x+x} \ln^2 G(t) dt < 0$  for all  $x_i \geq 0$ . Let

$$E\pi_n(x, \hat{y}) = \int_y^{\hat{y}} G(t)^{(n-1)x} H(t | x) dt - C_1 x - C_2 \quad (4)$$

be the firm's ex ante expected profit given the symmetric equilibrium strategies of both investment and bidding. Each potential firm enters the R&D process if its expected profit is nonnegative. That is, equilibrium entry gives

$$E\pi_n(x, \hat{y}) \geq 0. \quad (\text{EEa})$$

And any additional entrant  $n+1$  earns negative profits:

$$E\pi_{n+1}(x', \hat{y}') < 0, \quad (\text{EEb})$$



where  $x'$  and  $y'$  are the individual R&D expenditure and the break-even cost level which are determined by (FE) and (R&D) when  $n + 1$  firms simultaneously enter the R&D process. Since the equilibrium bidding strategy  $B(y)$  is completely determined by  $(n, x, y)$ , we only have to consider  $(n, x, y)$  for a symmetric perfect free-entry equilibrium. Therefore, for any given reservation price  $r$ , equations (3), (FE), (EEa), (EEb), and (R&D) *simultaneously* determine the symmetric perfect free-entry equilibrium  $(n_e, x_e, y_e)$  with the bidding function in (3), where  $n_e$  is the equilibrium number of informed firms,  $x_e$  is the each firm's equilibrium investment level in R&D, and  $y_e$  is the break-even cost level that the informed firm will bid if its cost is no higher than  $y_e$ . The total expenditure in R&D at the equilibrium is  $n_e x_e$ , denoted by  $\bar{x}_e$ . At the equilibrium, each informed firm invests  $x_e$  in R&D and the firms with cost observations higher than  $y$  submit bids. The number of firms that actually submit bids is random and subject to a binomial distribution. Thus, the average (or expected) number of actual bidders is  $n_a = n_e H(y_e | x_e)$ , which depends on the equilibrium number of informed firms, the investment level in R&D, and the break-even cost level. When there is a bid preparation cost,  $y_e < \bar{y}$  and hence the average number of actual bidders is less than the number of informed firms.

I need to show the existence and uniqueness of symmetric perfect free-entry equilibrium for a given reservation price. I first consider the special case where there is no bid preparation cost. Then, from (FE),  $y$  is the same as the reservation price  $r$ , and (R&D) is a one variable equation which determines the total investment level  $\bar{x}_e$ . Substituting  $\bar{x}_e$  into (EEa) and (EEb), it should be easy to solve for the individual investment level  $x_e$  and the number of firms  $n_e$ . I allow the number of firms  $n$  to be a continuous variable at this moment and adjust the solution later on. Each firm enters the R&D process until its expected profit is zero and hence the equilibrium-entry conditions (EEa) and (EEb) can be represented by the equality

$$\int_{\underline{y}}^y G(t)^{(n-1)x} H(t|x) dt - C_1 x - C_2 = 0. \quad (\text{EE})$$

Then the following are true:

**Proposition 1:** In the case of  $K = 0$ , there exists a unique solution  $(n_e, x_e, y_e)$ , with  $n_e \in (0, +\infty)$ ,  $x_e \in (0, +\infty)$ , and  $y_e = r$ , to the system of equations (FE), (EE), and (R&D) if and only if  $C_2 > 0$  and  $0 < C_1 < -\int_{\underline{y}}^r \ln G(t) dt$ . Furthermore, (a)  $\frac{\partial \bar{x}_e}{\partial r} > 0$  and  $\frac{\partial \bar{x}_e}{\partial C_1} < 0$ ; (b)  $\frac{\partial \bar{x}_e}{\partial C_2} = 0$ ,  $\frac{\partial n_e}{\partial C_2} < 0$ ,  $\frac{\partial x_e}{\partial C_2} > 0$ , and  $\frac{\partial n_a}{\partial C_2} < 0$ .

Proof: Since  $K = 0$ , by definition  $y_e = r$ . Then (R&D) and (EE) form a recursive system. Let

$$\phi(\bar{x}) \equiv \int_{\underline{y}}^r G(t)^{\bar{x}} \ln G(t) dt + C_1,$$

then  $\phi(0) = \int_{\underline{y}}^r \ln G(t) dt + C_1$ ,  $\phi(+\infty) = C_1$ , and  $\phi'(\bar{x}) > 0$  for all  $\bar{x} > 0$ . By continuity,  $\phi_1(\bar{x})$  has a unique positive root and hence condition (R&D) uniquely determines a solution  $0 < \bar{x}_e = n_e x_e < +\infty$  if and

only if  $0 < C_1 < -\int_{\underline{y}}^r \ln G(t) dt$ . Similarly, let

$$\psi(x) \equiv \int_{\underline{y}}^r G(t)^{\bar{x}-x} dt - \int_{\underline{y}}^r G(t)^{\bar{x}} dt - C_1 x - C_2,$$

then  $\psi(0) = -C_2$ ,  $\psi(+\infty) = +\infty$ , and  $\psi'(x) > 0$  for all  $x > 0$  because of equation (R&D). Then by continuity,  $\psi(x)$  has a unique positive root  $x_e$  if and only if  $\psi(0) = -C_2 < 0$ , i.e.  $C_2 > 0$ . Let  $n_e = \bar{x}_e / x_e$ . Therefore, (FE), (EE), and (R&D) determine a unique solution  $(n_e, x_e, \hat{y}_e)$  with  $0 < n_e < +\infty$ ,  $0 < x_e < +\infty$ , and  $\hat{y}_e = r$ .

Now, taking the derivatives of both sides of equation (R&D) with respect to  $\hat{y}_e = r$ , I have

$$G(r)^{\bar{x}} \ln G(r) + \frac{\partial \bar{x}_e}{\partial r} \int_{\underline{y}}^r G(t)^{\bar{x}} \ln^2 G(t) dt = 0.$$

Since  $0 < G(t) < 1$  for all  $t \in (\underline{y}, \bar{y})$ , we have  $\frac{\partial \bar{x}_e}{\partial r} > 0$  for any  $r \in (\underline{y}, \bar{y})$ . Taking the derivatives of both sides of equation (R&D) at the free-entry equilibrium with respect to  $C_1$ , I get

$$\frac{\partial \bar{x}_e}{\partial C_1} \int_{\underline{y}}^r G(t)^{\bar{x}} \ln^2 G(t) dt + 1 = 0$$

which obviously implies  $\frac{\partial \bar{x}_e}{\partial C_1} < 0$ .

Similarly, I can show (b) by taking the derivatives of both sides of equation (EE) and (R&D) at the free-entry equilibrium with respect to  $C_2$ .

Q.E.D.

From Proposition 1, the higher the marginal cost of expenditure on R&D is, the lower the total expenditure is. The condition  $C_1 < -\int_{\underline{y}}^r \ln G(t) dt$  in Proposition 1 is required so that the total expenditure on R&D is positive. That is, in order to have some R&D activity in the industry, the marginal cost of R&D cannot be too high. On the other hand, when the marginal cost of R&D approaches zero, the total expenditure on R&D approaches infinity no matter how large the fixed cost of R&D is.

The fixed cost of R&D is sunk and does not affect the total expenditure, but does affect the equilibrium number of informed firms and the average number of actual bidders as well. When this fixed cost decreases, the equilibrium number of firms increases. In the limit, as the fixed cost of R&D approaches zero, the equilibrium number of informed firms  $n_e$  approaches infinity and each firm invests almost zero in R&D. To avoid this limit case, the buyer could introduce a positive entry fee that each firm would pay prior to undertaking an expenditure in R&D. The higher the entry fee is, the less the number of firms is in the equilibrium.

If  $n_e$  is an integer, each informed firm gets exactly zero expected profit at the equilibrium. If  $n_e$  is not an integer, the equilibrium needs to be adjusted. Let  $[n_e]$  represents the largest integer which is less than or equal to  $n_e$ . If  $[n_e]$  firms become active, the total expenditure on R&D does not change. Then each firm invests  $x'_e = \bar{x}_e / [n_e]$  on R&D and  $x'_e > x_e$ . Since  $E\pi_{n_e}(x_e, r) = 0$  from (4) and (EE), we have

$$\begin{aligned} E\pi_{[n_e]}(x'_e, r) - E\pi_{n_e}(x_e, r) &= \int_{\underline{y}}^r G(t)^{\bar{x}_e - x'_e} dt - \int_{\underline{y}}^r G(t)^{\bar{x}_e - x_e} dt - C_1(x'_e - x_e) \\ &= \frac{1}{2}(x'_e - x_e)^2 \int_{\underline{y}}^r G(t)^{\bar{x}_e - \xi} \ln^2 G(t) dt > 0, \end{aligned}$$

where  $x'_e \geq \xi \geq x_e$  and the second equality holds because of Taylor's expansion and equation (R&D). Thus each firm earns a positive expected profit. The above expression can also be used to estimate how much expected economic profits each firm is able to earn.

On the other hand, if more than  $[n_e]$  firms become active, each firm would invest  $x$  on R&D which is strictly less than  $x_e$ . A similar argument implies that each firm would earn a negative profit. Thus,  $([n_e], x'_e, r)$  with (3) is the correct symmetric perfect free-entry equilibrium in this case.

In more general cases where  $K > 0$ , I am also able to show the existence and uniqueness of a symmetric perfect free-entry equilibrium. For any given number of firms that invest in R&D, let us first look at the firms' R&D behavior and exit decisions. Let  $\alpha = n$  be a continuous variable parameter,  $\alpha \geq 1$ . Given any  $\alpha$ , consider the solution  $(x_\alpha, \hat{y}_\alpha)$  to the equations system (FE) and (R&D) and let  $\pi(\alpha) \equiv E\pi_\alpha(x_\alpha, \hat{y}_\alpha)$  be each firm's expected profit for a given reservation price  $r$  when there are  $\alpha$  firms becoming active. Also let  $\bar{x}_\alpha = \alpha x_\alpha$ . Then I have

**Proposition 2:** In the case of  $K > 0$ , suppose  $\underline{y} < r - K \leq \bar{y}$ , and  $0 < C_1 < -\int_{\underline{y}}^{r-K} \ln G(t) dt$ . Then for any  $\alpha \in [1, +\infty)$  there exists a unique solution  $(x_\alpha, \hat{y}_\alpha)$ , with  $x_\alpha > 0$  and  $\hat{y}_\alpha \in (\underline{y}, r - K)$ , to the equations system (FE) and (R&D). Furthermore,

- (a)  $x_\alpha, \bar{x}_\alpha, \hat{y}_\alpha$ , and  $\pi(\alpha)$  are all continuous and strictly decreasing in  $\alpha \in (1, +\infty)$ ;
- (b)  $\hat{y}_\alpha \rightarrow \hat{y}_\infty \in (\underline{y}, r - K)$ ,  $x_\alpha \rightarrow 0$ ,  $\bar{x}_\alpha \rightarrow \bar{x}_\infty > 0$ , and  $\pi(\alpha) \rightarrow -C_2$  when  $\alpha \rightarrow +\infty$ ;
- (c)  $\hat{y}_\alpha \rightarrow r - K$ ,  $x_\alpha \rightarrow x_1$ ,  $\bar{x}_\alpha \rightarrow \bar{x}_1$ , and  $\pi(\alpha) \rightarrow \pi(1)$  when  $\alpha \rightarrow 1$ .

**Proof:** First of all, we show the existence and uniqueness of the solution to (FE) and (R&D). If  $\alpha = 1$ , then, from (FE),  $\hat{y}_1 = r - K$ . Equation (R&D) determines a unique solution  $x_1 > 0$  since  $0 < C_1 < -\int_{\underline{y}}^{r-K} \ln G(t) dt$  by the assumption.

If  $\bar{\alpha} > 1$ , then  $r - \hat{y} > 0$  from (FE) since  $K > 0$ . Then (FE) gives

$$\phi(\hat{y}) \equiv (\alpha - 1)x = \frac{\ln K - \ln(r - \hat{y})}{\ln G(\hat{y})} \quad (5)$$

which also implies  $\phi(\underline{y}) = +\infty$ ,  $\phi(r - K) = 0$ , and  $\phi'(\hat{y}) < 0$  for any  $\hat{y} \in (\underline{y}, r - K)$ . Substitute (5) into

equation (R&D) and let

$$\psi(\underline{y}) \equiv \int_{\underline{y}}^{\hat{y}} G(t)^{\alpha\phi(\underline{y})/(\alpha-1)} \ln G(t) dt + C_1.$$

Then  $\psi(\underline{y}) = C_1 > 0$ ,  $\psi(r - K) = \int_{\underline{y}}^{r-K} \ln G(t) dt + C_1 < 0$  by the assumption, and

$$\psi'(\underline{y}) = G(\underline{y})^{\alpha\phi(\underline{y})/(\alpha-1)} \ln G(\underline{y}) + \frac{\alpha}{\alpha-1} \phi'(\underline{y}) \int_{\underline{y}}^{\hat{y}} G(t)^{\alpha\phi(\underline{y})} \ln^2 G(t) dt < 0$$

for any  $\underline{y} \in (\underline{y}, r - K)$ . By the continuity of  $\psi(\underline{y})$ , there exists a unique root  $\hat{y}_\alpha$  of  $\psi$ . Substituting  $\hat{y} = \hat{y}_\alpha$  into (5), we can calculate  $x_\alpha = \phi(\hat{y}_\alpha) / (\alpha - 1) > 0$ . Thus, for  $\alpha > 1$ , there exists a unique solution  $(x_\alpha, \hat{y}_\alpha)$  to the equations system (FE) and (R&D) with  $x_\alpha > 0$  and  $\hat{y}_\alpha \in (\underline{y}, r - K)$ .

Second, we prove that (a) holds. It is easy to see  $x_\alpha$ ,  $\bar{x}_\alpha$ ,  $\hat{y}_\alpha$ , and  $\pi(\alpha)$  are all continuously differentiable in  $\alpha \in (1, +\infty)$ . Taking the derivatives of both sides of (FE) and (R&D) with respect to  $\alpha$ , respectively, we obtain

$$\rho(\hat{y}_\alpha) \frac{\partial \hat{y}_\alpha}{\partial \alpha} + \left[ (\alpha - 1) \frac{\partial x_\alpha}{\partial \alpha} + x_\alpha \right] K \ln G(\hat{y}_\alpha) = 0 \quad (6)$$

and

$$G(\hat{y}_\alpha)^{(\alpha-1)x_\alpha} \ln G(\hat{y}_\alpha) \frac{\partial \hat{y}_\alpha}{\partial \alpha} + \left[ \alpha \frac{\partial x_\alpha}{\partial \alpha} + x_\alpha \right] \int_{\underline{y}}^{\hat{y}_\alpha} G(t)^{\alpha x_\alpha} \ln^2 G(t) dt = 0, \quad (7)$$

where

$$\rho(\hat{y}) = \frac{\partial}{\partial \hat{y}} \left[ (r - \hat{y}) G(\hat{y})^{(\alpha-1)x} \right] = -G(\hat{y})^{(\alpha-1)x} - x(\alpha-1)(r - \hat{y}) G(\hat{y})^{(\alpha-1)x-1} f(\hat{y}) < 0$$

for all  $\hat{y} \in (\underline{y}, r - K)$ . From (6) and (7), we can calculate  $\frac{\partial \hat{y}_\alpha}{\partial \alpha}$  as the following:

$$\begin{aligned} & \left[ (\alpha - 1) K G(\hat{y}_\alpha)^{(\alpha-1)x_\alpha} \ln^2 G(\hat{y}_\alpha) - \alpha \rho(\hat{y}_\alpha) \int_{\underline{y}}^{\hat{y}_\alpha} G(t)^{\alpha x_\alpha} \ln^2 G(t) dt \right] \frac{\partial \hat{y}_\alpha}{\partial \alpha} \\ & = x_\alpha K \ln G(\hat{y}_\alpha) \int_{\underline{y}}^{\hat{y}_\alpha} G(t)^{\alpha x_\alpha} \ln^2 G(t) dt \end{aligned}$$

which clearly implies  $\frac{\partial \hat{y}_\alpha}{\partial \alpha} < 0$  for all  $\alpha > 1$ . Then from (7) we know  $\frac{\partial x_\alpha}{\partial \alpha} < 0$  and  $\frac{\partial \bar{x}_\alpha}{\partial \alpha} < 0$  for all

$\alpha > 1$ . At the same time, from (6), we get  $\frac{\partial(\alpha-1)x_\alpha}{\partial \alpha} > 0$  for  $\alpha > 1$ .

Using equations (4) and (R&D), we can calculate

$$\frac{d\pi(\alpha)}{d\alpha} = \frac{\partial \hat{y}_\alpha}{\partial \alpha} G(\hat{y}_\alpha)^{\bar{x}_\alpha - x_\alpha} H(\hat{y}_\alpha | \bar{x}_\alpha) + \frac{\partial(\alpha-1)x_\alpha}{\partial \alpha} \int_{\underline{y}}^{\hat{y}_\alpha} G(t)^{\bar{x}_\alpha - x_\alpha} H(t | \bar{x}_\alpha) \ln G(t) dt.$$

The results we obtained above imply  $\frac{d\pi(\alpha)}{d\alpha} < 0$  for all  $\alpha > 1$ .

Third, let  $\alpha$  approach infinity. The equation  $\psi(\hat{y}_\alpha) = 0$  becomes

$$\int_{\underline{y}}^{\hat{y}_\alpha} G(t)^{\phi(\hat{y}_\alpha)} \ln G(t) dt + C_1 = 0, \quad (8)$$

which determines a unique solution  $\hat{y}_\alpha \in (\underline{y}, r - K)$ . Then (5) implies  $(\alpha - 1)x_\alpha \rightarrow \phi(\hat{y}_\alpha)$ ,  $x_\alpha \rightarrow 0$ , and  $\bar{x}_\alpha \rightarrow \phi(\hat{y}_\alpha) \equiv \bar{x}_\infty > 0$  when  $\alpha \rightarrow +\infty$ . Using these results, we can easily see  $\pi(\alpha) \rightarrow -C_2$  when  $\alpha \rightarrow +\infty$ .

Finally, we prove (c). Since  $\hat{y}_\alpha$  is continuous and strictly decreasing in  $\alpha$  for all  $\alpha > 1$  and has an upper bound  $r - K$ , then  $\hat{y}_\alpha$  has a limit when  $\alpha$  approaches 1, denoted by  $\hat{y}_0$  with  $\hat{y}_0 \leq r - K$ . Suppose  $\hat{y}_0 < r - K$ . In the following, we can show that there exists  $\tilde{\alpha} > 1$  such that  $\hat{y}_{\tilde{\alpha}} = \hat{y}_0$ . Then  $\hat{y}_\alpha > \hat{y}_0$  for all  $\alpha \in (1, \tilde{\alpha})$  and hence  $\hat{y}_0$  cannot be the limit of  $\hat{y}_\alpha$ . This is a contradiction. Thus,  $\hat{y}_0 = r - K$ .

In fact, if  $\hat{y}_0 < r - K$  then  $\phi(\hat{y}_0) > 0$ , where  $\phi(\hat{y})$  is defined by (5). Let

$$v(\alpha) \equiv \int_{\underline{y}}^{\hat{y}_\alpha} G(t)^{\alpha\phi(\hat{y}_\alpha)(\alpha-1)} \ln G(t) dt + C_1,$$

Then  $v'(\alpha) < 0$  for all  $\alpha > 1$  and  $v(\alpha) \rightarrow C_1 > 0$  when  $\alpha \rightarrow 1$ . Since  $\hat{y}_0 > \hat{y}_\infty$  and  $\phi(\hat{y}_0) < \phi(\hat{y}_\infty)$ , using (8), we get

$$\begin{aligned} v(+\infty) &= \int_{\underline{y}}^{\hat{y}_0} G(t)^{\phi(\hat{y}_0)} \ln G(t) dt + C_1 \\ &= \int_{\underline{y}}^{\hat{y}_0} G(t)^{\phi(\hat{y}_0)} \ln G(t) dt + \int_{\underline{y}}^{\hat{y}_\infty} \left[ G(t)^{\phi(\hat{y}_0)} - G(t)^{\phi(\hat{y}_\infty)} \right] \ln G(t) dt < 0. \end{aligned}$$

Thus, there exists  $\tilde{\alpha} > 1$  such that  $\psi(\tilde{\alpha}) = 0$ . Let  $x_{\tilde{\alpha}} = \tilde{\alpha}\phi(\hat{y}_0) / (\tilde{\alpha} - 1)$ , then  $(x_{\tilde{\alpha}}, \hat{y}_{\tilde{\alpha}})$  is the unique solution to (FE) and (R&D), where  $\hat{y}_{\tilde{\alpha}} = \hat{y}_0$ .

Since  $\hat{y}_0 = r - K$  and  $\phi(\hat{y}_0) = 0$ , equation (5) implies  $(\alpha - 1)x_\alpha \rightarrow 0$  when  $\alpha \rightarrow 1$ . From equation  $\psi(\hat{y}_\alpha) = 0$ , we know  $\hat{x}_\alpha \rightarrow x_1$  when  $\alpha \rightarrow 1$ . Thus, it is easy to see  $\pi(\alpha) \rightarrow \pi(1)$  when  $\alpha \rightarrow 1$ .

Q.E.D.

For any given number of active firms, the R&D expenditure and the break-even cost level are uniquely determined. When more firms become active, each active firm's expected profit

decreases. More competition makes the procurement contract less profitable to each firm. Each firm intends to invest less in R&D. The total expenditure on R&D among all firms is also lower. If there were no bid-preparation cost, the total R&D expenditure would not change with the number of active firms. More firms increase the total bid-preparation costs and discourage R&D over all.

When the number of active firms goes to infinity, each firm invests almost zero on R&D although the total R&D expenditure approaches a positive amount. Each firm's expected profit approaches  $-C_2$ . Thus if there is no fixed cost of R&D then free entry causes an infinite number of firms to enter the R&D process. If there is a positive fixed R&D cost then only a finite number of firms will decide to enter the R&D process. I will make this point more precise in the next proposition. Therefore, the fixed cost of R&D  $C_2$  is the key determinant of the free-entry equilibrium number of firms although the latter is also affected directly or indirectly by the marginal cost of R&D  $C_1$ , the bidding preparation cost  $K$ , the reservation price  $r$ , and the distribution of production cost  $H(y|x)$ .

If  $\pi(1) \leq 0$  for any reservation price  $r$ , no firm can make any profit from conducting R&D and production. This is not an interesting case. I assume  $\pi(1)$  is positive for a given reservation price  $r$ . That is, when there is only one firm participating in R&D activity and bidding for the procurement contract, that firm is able to earn positive profits. Under free entry, at least one firm will then enter the R&D and bidding process. From Proposition 2, each firm will earn a profit  $\pi(\alpha)$  which is strictly decreasing in the number of firms entered  $\alpha$ . Firms enter until this profit equals zero. If the fixed cost of R&D is positive, the equilibrium number of active firms should be finite. Formally, I have

**Proposition 3:** Suppose  $K$ ,  $C_1$ ,  $C_2$ ,  $\pi(1)$  are all positive and  $\underline{y} < r - K \leq \bar{y}$ . Then there exists a unique symmetric perfect free-entry equilibrium  $(n_e, x_e, \hat{y}_e)$  with (3), where the integer  $n_e \geq 1$ ,  $x_e > 0$ , and  $\underline{y} < \hat{y}_e < r - K$ .

**Proof:**<sup>9</sup> I first want to show that there exists a unique solution to (FE), (R&D), and (EE). This is equivalent to showing that there exists a unique  $\alpha \geq 1$  such that  $\pi(\alpha) = 0$ . Let

$$u(x) \equiv \int_{\underline{y}}^{r-K} H(t|x) dt - C_1 x - C_2$$

for  $x \geq 0$ , then  $\pi(1) = \max u(x)$ . The assumptions  $\pi(1) > 0$  and  $C_2 > 0$  imply that there exists  $x_1 > 0$  such that  $\pi(1) = u(x_1)$  and  $u'(x_1) = 0$ . Since  $u''(x) < 0$  for all  $x \geq 0$ , then  $u'(0) > u'(x_1) = 0$ . That is,  $C_1 < -\int_{\underline{y}}^{r-K} \ln G(t) dt$ . Thus, the assumptions in Proposition 2 are satisfied. According to Proposition 2,  $\pi(\alpha)$  is continuous and strictly decreasing over  $\alpha \in [1, +\infty)$  with  $\pi(+\infty) = -C_2 < 0$ . Then the assumption  $\pi(1) > 0$  implies that there is a unique  $\alpha^* > 1$  such that  $\pi(\alpha^*) = 0$ .

Let  $n_e = [\alpha^*]$  be the largest integer which is less than or equal to  $\alpha^*$ , and let  $x_e = x_{[\alpha^*]}$  and  $\hat{y}_e = \hat{y}_{[\alpha^*]}$ . Since  $[\alpha^*] \leq \alpha^* < [\alpha^*] + 1$ ,  $\pi(\alpha^*) = 0$ , and  $\pi(\alpha)$  is strictly decreasing in  $\alpha$ , then

$$E \pi_{n_e}(x_e, \hat{y}_e) = \pi([\alpha^*]) \geq 0$$

and

$$E\pi_{n_e+1} = \pi([\alpha^*] + 1) < 0.$$

Thus,  $(n_e, x_e, \hat{y}_e)$  satisfy (FE), (R&D), (EEa), and (EEb) with the integer  $n_e \geq 1$ ,  $x_e > 0$ , and  $\hat{y}_e \in (\underline{y}, r - K)$ . That is, with (3),  $(n_e, x_e, \hat{y}_e)$  is a unique symmetric perfect free-entry equilibrium.

Q.E.D.

#### 4. Optimality from the Buyer's Point of View

Now, go back to the first stage of the three-stage game and look at the buyer's optimality problem. I want to know whether there exists a reservation price under which the free-entry equilibrium characterized in the last section is optimal for the buyer.

For any given number of active firms, the distribution of production cost  $y$  of the winning firm is  $1 - [1 - H(y|x)]^n = 1 - G(y)^n$ . The buyer's ex ante expected costs in the competitive procurement are

$$\int_{\underline{y}}^{\hat{y}} B(y) d(1 - G(y)^n) = \int_{\underline{y}}^{\hat{y}} y d(1 - G(y)^n) + nKH(\hat{y}|x) + nC(x) + nE\pi_n(x, \hat{y}) \quad (9)$$

where  $E\pi_n(x, \hat{y})$  is a firm's expected profits under the symmetric equilibria, defined by (4). The buyer's expected costs in the competitive procurement with R&D include the expected minimum production cost, the total R&D costs among all firms, the total expected bid-preparation costs, and the total expected profits among all firms.

Under free entry, each firm enters the R&D and bidding processes until its expected profit  $E\pi_n(x, \hat{y})$  equals zero. Therefore, the winner's expected profits  $\int_{\underline{y}}^{\hat{y}} (B(y) - y) d(1 - G(y)^n)$  from the competitive bidding are equal to the total costs on both R&D and bid preparation among all of the firms. In other words, if free entry is allowed, the rents for the firms from contracting are dissipated by precontract R&D and bid-preparation activities. The question, as I will answer in this and the next sections, is whether these R&D activities are good for the buyer and society.

At the symmetric free-entry equilibrium under a given reservation price, what the buyer has to pay is not just the expected minimum production cost, but also the total R&D cost  $n_e(C_1x_e + C_2)$  of all informed firms, and the total bid preparation cost  $n_eK = n_eKH(\hat{y}_e|x_e)$  of all actual bidders as well. One might have thought that the buyer has only to pay the R&D costs of the winner. But since firms are assumed to be symmetric and to adopt the same investment and bidding strategy, each has an equal probability to be the winner. Therefore, the buyer actually ex ante expects to pay all of the costs of R&D among active firms.

Remember that the buyer can procure the good elsewhere at the cost  $y_0$  if the lowest bid is higher than the reservation price  $r$ . Because of the bid preparation cost,  $B(\hat{y}) = r$  and the firm with cost  $y$  bids if and only if  $y \leq \hat{y}$ . The buyer actually procures the good at cost  $y_0$  elsewhere with probability  $[1 - H(\hat{y}|x)]^n = G(\hat{y})^n$ . Thus, the buyer's total expected costs will be, remember

$\hat{y} = \hat{y}(r, n, x)$  from (FE),

$$\begin{aligned}
 EBC(r, n, x) &= \int_{\underline{y}}^{\hat{y}} B(y) d(1 - G(y)^{nx}) + y_0 G(\hat{y})^{nx} \\
 &= \underline{y} + (y_0 - \hat{y}) G(\hat{y})^{nx} + nx - nx G(\hat{y})^x \\
 &\quad + n \int_{\underline{y}}^{\hat{y}} G(t)^{(n-1)x} dt - (n-1) \int_{\underline{y}}^{\hat{y}} G(t)^{nx} dt.
 \end{aligned} \tag{10}$$

The buyer wants to minimize his total ex ante expected costs of procurements  $EBC(r, n, x)$  by selecting  $r, n$ , and possibly  $x$ . Since the buyer has to pay all the costs in (9), as a tradeoff, he may want to set  $\hat{y}$  less than his opportunity cost  $y_0$ .

I first consider a fixed-scale R&D technology. That is, each firm either invests in R&D at a cost  $C > 0$  or does not invest. If the firm invests in R&D, it observes its production  $y$  and believes that other investing firms' production cost observations are independently drawn from the same cumulative distribution  $F(y)$  with the support  $[\underline{y}, \bar{y}]$ . I also assume that there is no bid preparation cost before the competitive bidding; then  $\hat{y} = r$  from (FE). Thus, the buyer's expected cost (10) can simply be written as

$$EBC(r, n) = \underline{y} + (y_0 - r) G(r)^n + \int_{\underline{y}}^r G(t)^n dt + n \int_{\underline{y}}^r F(t) G(t)^{n-1} dt. \tag{11}$$

Suppose  $C = 0$ . That is, each potential firm can observe its own production cost  $y$  without any expense and believes that other firms' production costs are drawn independently from the same distribution  $F(y)$ . The buyer then chooses  $(r, n)$  to maximize his expected profit (11). It can be easily shown that the following are true: First, the buyer should choose the optimal reservation price  $r = r_0$  such that

$$r_0 + \frac{F(r_0)}{f(r_0)} = y_0 \tag{12}$$

if  $y_0 < \bar{y} + 1/f(\bar{y})$  and  $r_0 = \bar{y}$  otherwise (see also Riley and Samuelson 1981 for a proof). The optimal reservation price  $r_0$  for the buyer in (12) is independent of the number of firms and is strictly less than his opportunity cost  $y_0$ . It is possible that the buyer procures somewhere else at cost  $y_0$  even though the winner in the competitive bidding offers a lower cost than  $y_0$ . Thus, because of asymmetry of information between the buyer and firms, the buyer finds it in his interest to distort the outcome away from the efficient allocation. Second, since  $r_0$  is independent of  $n$ ,  $EBC(r_0, n) \geq \underline{y}$ , and  $EBC(r_0, n)$  goes to  $\underline{y}$  when  $n$  approaches infinity, the buyer prefers an infinite number of firms to bid for the contract. Since each firm has a positive expected profit  $\int_{\underline{y}}^r F(t) G(t)^{n-1} dt$ , free entry will cause an infinite number of firms in the competitive bidding process and drive the production cost to the lowest bound  $\underline{y}$ . Therefore, the buyer prefers free entry in this case.



Now, suppose  $C > 0$ . In order to become informed about production costs, each firm has to pay a R&D cost  $C > 0$  before the competitive bidding. This is the case considered by McAfee and McMillan (1987). But in their model, they assume that the opportunity cost of the procurement for the buyer is so high that no reservation is needed. As we will see in the following, if the opportunity cost is relatively low, a reservation price is necessary for the buyer. The buyer chooses  $r \in [\underline{y}, \bar{y}]$  and  $n \geq 1$  to minimize his expected costs (11) subject to each firm's nonnegative profits constraint (EEa):

$$E\pi_n(r) \equiv \int_{\underline{y}}^r F(t)G(t)^{n-1}dt - C \geq 0. \quad (13)$$

Consider  $n$  as a real variable. It is easy to see  $EBC(r, n)$  and  $E\pi_n(r)$  are all continuous functions with respect to  $r$  and  $n$ . Since  $E\pi_n(r)$  is increasing in  $r$  and decreasing in  $n$ , the constraint (13) with  $r \in [\underline{y}, \bar{y}]$  and  $n \geq 1$  forms a non-empty compact set in  $R_+^2$  if  $C < \int_{\underline{y}}^{\bar{y}} F(t)dt$ . Thus, there exists a solution to the buyer's optimization problem. Let  $r^*$  and  $n^*$  be the buyer's optimal reservation price and the optimal number of firms, respectively. Then we have

**Proposition 4:** Under a fixed-scale R&D technology, if  $0 < C < \int_{\underline{y}}^{\bar{y}} F(t)dt$ , then i)  $r^* = \bar{y}$  when  $y_0 \geq \bar{y} + 1/f(\bar{y})$  and  $r_0 < r^* < y_0$  when  $y_0 \leq \bar{y}$ ; and ii) free entry causes the buyer's optimal number of firms  $n^*$  to enter the competitive procurement process.

**Proof:** The first order conditions of the buyer's optimization problem give the following:

$$\phi_\lambda(r, n) \equiv -\frac{\partial EBC}{\partial r} + \lambda \frac{\partial E\pi_n}{\partial r} = 0,$$

$$\psi_\lambda(r, n) \equiv -\frac{\partial EBC}{\partial n} + \lambda \frac{\partial E\pi_n}{\partial n} = 0,$$

and  $\lambda E\pi_n(r) = 0$  for interior solution  $r = r^* \in (\underline{y}, \bar{y})$  and  $n = n^* \in (1, +\infty)$ , where  $\lambda = \lambda^* \geq 0$  is the multiplier for the inequality constraint (13), and

$$\phi_\lambda(r, n) = nG(r)^{n-1}f(r) \left[ y_0 - r - \left(1 - \frac{\lambda}{n}\right) \frac{F(r)}{f(r)} \right]$$

$$\psi_\lambda(r, n) = -(y_0 - r)G(r)^n \ln G(r)$$

$$- \int_{\underline{y}}^r G(t)^{n-1} \left[ F(t) + \left[ 1 + (n-1-\lambda)F(t) \right] \ln G(t) \right] dt.$$

First, consider the case  $y_0 \geq \bar{y} + 1/f(\bar{y})$ . For any  $r < \bar{y}$  and  $n \geq 1$ , we have

$$\phi_\lambda(r, n) \geq nG(r)^{n-1}f(r) \left[ \bar{y} + \frac{1}{f(\bar{y})} - r - \frac{F(r)}{f(r)} + \frac{\lambda F(r)}{nf(r)} \right] > 0$$

since  $r + F(r)/f(r)$  is increasing in  $r$ . Thus,  $r^* = \bar{y}$ . We claim  $n^* > 1$  in this case. In fact, if  $n^* = 1$  then the first order condition gives  $\psi_{\lambda^*}(\bar{y}, 1) \leq 0$ . Since  $E\pi_1(\bar{y}) > 0$  by the assumption and  $\lambda^* E\pi_1(\bar{y}) = 0$ , we have  $\lambda^* = 0$ . But  $\psi_0(\bar{y}, 1) = -\int_{\bar{y}}^{\bar{y}} [F(t) + \ln G(t)] dt > 0$ . This contradicts with  $\psi_{\lambda^*}(\bar{y}, 1) \leq 0$ . Therefore  $n^* > 1$ . This with the first order condition implies  $\psi_{\lambda^*}(\bar{y}, n^*) = 0$ . Since  $\psi_0(\bar{y}, n^*) > 0$ ,  $\psi_n(\bar{y}, n^*) < 0$ , and  $\frac{d\psi_{\lambda^*}(\bar{y}, n^*)}{d\lambda} < 0$  for all  $\lambda > 0$ , equation  $\psi_{\lambda^*}(\bar{y}, n^*) = 0$  implies  $\lambda^* > 0$ . Thus,  $\lambda^* E\pi_{n^*}(\bar{y}) = 0$  implies  $E\pi_{n^*}(\bar{y}) = 0$  which determines a unique  $n^* > 1$ . Then  $\psi_{\lambda^*}(\bar{y}, n^*) = 0$  uniquely determines  $\lambda^* \in (0, n^*)$ . Therefore,  $r^* = \bar{y}$  and  $n^*$  determined by  $E\pi_{n^*}(\bar{y}) = 0$  are optimal for the buyer.

Second, consider the case  $y_0 \leq \bar{y}$ . If  $n^* = 1$ , then the first order condition implies  $\psi_{\lambda^*}(r^*, 1) \leq 0$ . Suppose  $r^* = \bar{y}$ , then  $\phi_{\lambda^*}(\bar{y}, 1) = f(\bar{y})(y_0 - \bar{y}) - (1 - \lambda^*) \geq 0$  which implies  $\lambda^* > 0$ . Then the first order condition  $\lambda^* E\pi_1(\bar{y}) = 0$  implies  $E\pi_1(\bar{y}) = 0$ . This contradicts with the assumption. Thus it must be the case  $r^* < \bar{y}$ . Then  $\phi_{\lambda^*}(r^*, 1) = 0$  holds, that is,

$$y_0 = r^* + (1 - \lambda^*) \frac{F(r^*)}{f(r^*)}. \quad (14)$$

Suppose  $r^* \geq y_0$ , then  $E\pi_1(r^*) \geq E\pi_1(y_0) > 0$  by the assumption. Thus  $\lambda^* E\pi_1(\bar{y}) = 0$  implies  $\lambda^* = 0$  and hence (14) implies  $y_0 > r^*$ . This contradicts to  $r^* \geq y_0$ . Therefore,  $r^* < y_0$ . Then the first order condition

$$0 \geq \psi_{\lambda^*}(r^*, 1) = -(y_0 - r^*)G(r^*)\ln G(r^*) - \int_{\bar{y}}^{r^*} \left[ F(t) + \left[ 1 - \lambda^* F(t) \right] \ln G(t) \right] dt$$

implies  $\lambda^* > 0$ . We know  $r^* > r_0$  from (14), where  $r_0$  is determined by (12). In summary, we have shown that if  $n^* = 1$  then  $\lambda^* > 0$  and  $r_0 < r^* < y_0$ .

If  $n^* > 1$ , then the first order condition gives  $\psi_{\lambda^*}(r^*, n^*) = 0$ . For  $n^* > 1$  and any  $r > \bar{y}$ , equation  $\psi_\lambda(r, n^*) = 0$  determines a unique  $\lambda = \lambda(r) \in (0, n^*)$  which is continuous at  $r = \bar{y}$ , and  $\lambda(\bar{y}) \in (0, n^*)$ . Because of the inequality  $\bar{y} + (1 - \frac{\lambda(\bar{y})}{n^*}) \frac{1}{f(\bar{y})} > \bar{y}$  and the continuity, we have

$$r + (1 - \frac{\lambda(r)}{n^*}) \frac{F(r)}{f(r)} > \bar{y}$$

and

$$\phi_{\lambda(r)}(r, n^*) < n^* G(r)^{n^*-1} f(r) (y_0 - \bar{y}) \leq 0$$

when  $r$  is close enough to  $\bar{y}$  and  $r < \bar{y}$ . Thus, it must be the case  $r^* < \bar{y}$ . Then  $\phi_{\lambda^*}(r^*, n^*) = 0$  holds, that is,

$$y_0 = r^* + (1 - \frac{\lambda^*}{n^*}) \frac{F(r^*)}{f(r^*)}. \quad (15)$$

Combining (15) with  $\psi_{\lambda^*}(r^*, n^*) = 0$ , we obtain  $\lambda^* = \lambda(r^*) \in (0, n^*)$ . Then

$$E \pi_n(r^*) = \int_{\underline{y}}^{r^*} F(t) G(t)^{n^*-1} dt - C = 0 \quad (16)$$

and  $r_0 < r^*$ . In other words, equations (15), (16), and  $\psi_{\lambda^*}(r^*, n^*) = 0$  simultaneously determine  $r^*$ ,  $n^*$ , and  $\lambda^*$  with  $0 < \lambda^* < n^*$  and  $r_0 < r^* < y_0$ .

I have shown  $\lambda^* > 0$  in both cases. That is, the firm's nonnegative profits constraint (13) is binding. Therefore, if free entry is allowed, the buyer's optimal number of firms  $n^*$  enter the R&D process provided that the buyer chooses the optimal reservation price  $r^*$ .

Q.E.D.

If  $n^*$  is not an integer, then similar to the discussion in the last section,  $[n^*]$  will be the optimal number of firms for the buyer. Each of  $[n^*]$  firms earns a positive expected profit.

The condition  $C < \int_{\underline{y}}^{y_m} F(t) dt$  in Proposition 4 is equivalent to  $E \pi_1(y_m) > 0$  which means that, under the highest reservation price  $y_m$ , if only one firm conducts R&D and production, that firm earns a positive expected profit. In other words, conducting R&D and production is potentially profitable. Otherwise, there is no interest in analyzing the optimal policy for the buyer or society.

In a competitive procurement with a fixed cost of R&D, if the buyer's opportunity cost  $y_0$  is relatively high, no reservation price is needed and the optimal number of firms enter the procurement process. That is the same as the result obtained by McAfee and McMillan (1987). In addition, they show that the sealed-bid auction without reservation price is an optimal mechanism.

If the opportunity cost  $y_0$  is relatively low (lower than the highest possible production cost level  $\bar{y}$ ), however, the optimal number of potential firms still enter the procurement process provided that the buyer chooses an optimal reservation price  $r^*$  which is lower than the buyer's opportunity cost  $y_0$ . The optimal reservation price  $r^*$  is higher than the reservation price  $r_0$  in the case where no such R&D cost exists. Thus, the distortion of the efficient outcome (see Section 5) due to asymmetry of information still exists, but the positive R&D cost reduces that distortion. The fact that each firm has to pay a positive cost to become informed reduces the asymmetry of information between the buyer and firms compared to the usual adverse selection models. I have also made a similar argument in Tan (1989).

Now, consider a variable scale R&D process subject to constant marginal returns to scale on R&D expenditure, where expenditure  $x$  is an endogenous continuous variable. Suppose that the buyer is able to control the firm's R&D decision and treat  $x$  as observable. Thus the buyer can control  $r$ ,  $n$ , and  $x$ . Suppose that there is no bid-preparation cost, then the buyer wants to choose

$(r, n, x)$  to solve his following optimization problem:

$$\begin{aligned} \text{Min } EBC(r, n, x) = & \underline{y} + (y_0 - r)G(r)^{nx} + n \int_{\underline{y}}^r G(t)^{(n-1)x} dt - (n-1) \int_{\underline{y}}^r G(t)^{nx} dt \\ & r, n, x \end{aligned} \quad (17)$$

$$\text{s.t. } E\pi_n(r, x) = \int_{\underline{y}}^r G(t)^{(n-1)x} H(t|x) dt - C_1 x - C_2 \geq 0$$

for  $r \in [\underline{y}, \bar{y}]$ ,  $n \geq 1$ , and  $x \geq 0$ . As before, I treat  $n$  as a real variable. Since  $EBC(r, n, x)$  is continuous and the constraints form a compact set, there exists a solution to the above optimization problem (17). Would the buyer still be satisfied with the symmetric free-entry equilibrium with a reservation price as I characterized in the last section? In other words, would the buyer give each firm freedom to make decisions on R&D and entry even though he can control them?

Let  $E\pi_1(r) = \max E\pi_1(r, x)$  over  $x \in [0, +\infty)$  be the expected profit when there is only one firm to conduct R&D and to make a bid under a buyer's reservation price  $r$ . It is easy to see  $E\pi_1(r)$  is increasing in  $r$ . I assume  $E\pi_1(y_m) > 0$ , that is, at the highest possible reservation price  $r = y_m$ , the sole firm that does both R&D and production should earn a positive expected profit. Then I have

**Proposition 5:** Suppose  $C_1$ ,  $C_2$ , and  $E\pi_1(y_m)$  are all positive, then there does not exist a reservation price under which the symmetric free-entry equilibrium solves the buyer's optimization problem (17).

**Proof:** Since  $C_1 > 0$  and  $C_2 > 0$  by the assumption, from the constraint of (17),  $x = 0$  and  $x = +\infty$  cannot be solutions to (17), nor  $r = \underline{y}$  and  $n = +\infty$ . A necessary condition for the optimal interior solution  $(r, n, x)$  to (17) is that there exist  $\lambda \geq 0$  such that

$$nxf(r)G(r)^{nx-1} \left[ y_0 - r - \left(1 - \frac{\lambda}{n}\right) \frac{H(r|x)}{h(r|x)} \right] \geq 0, \quad (18)$$

$$\begin{aligned} & -x(y_0 - r)G(r)^{nx} \ln G(r) - \int_{\underline{y}}^r G(t)^{(n-1)x} \phi(t|x) dt \\ & + (\lambda - n)x \int_{\underline{y}}^r G(t)^{(n-1)x} H(t|x) \ln G(t) dt \leq 0, \end{aligned} \quad (19)$$

$$\begin{aligned} & -n(y_0 - r)G(r)^{nx} \ln G(r) + (\lambda - n)(n-1) \int_{\underline{y}}^r G(t)^{(n-1)x} H(t|x) \ln G(t) dt \\ & + \lambda \left[ - \int_{\underline{y}}^r G(t)^{nx} \ln G(t) dt - C_1 \right] = 0, \end{aligned} \quad (20)$$

and  $\lambda E \pi_n(r, n) = 0$ , where  $\phi(t|x) = 1 - G(t)^x + xG(t)^x \ln G(t) > 0$  for all  $t > \underline{y}$ . Suppose  $n > 1$ . If  $r = \bar{y}$  then (19) with equality implies  $0 < \lambda < n$  which with (20) implies

$$-\int_{\underline{y}}^{\bar{y}} G(t)^{nx} \ln G(t) dt - C_1 < 0. \quad (21a)$$

If  $r < \bar{y}$  then substituting (18) into (19) with both equality we observe  $\lambda < n$ . Substituting (18) with equality into (20) and using  $0 \leq \lambda < n$ , we also obtain

$$-\int_{\underline{y}}^r G(t)^{nx} \ln G(t) dt - C_1 < 0. \quad (21b)$$

Therefore, any solution  $(r, n, x)$  with  $n > 1$  to (17) will violate the equilibrium condition (R&D).

On the other hand, suppose  $n = 1$  is a solution to (17) and satisfies equation (R&D), then (20) becomes

$$(y_0 - r)G(r)^x \ln G(r) = 0.$$

This together with (18) implies  $r = y_m$ . Then (19) becomes

$$-\int_{\underline{y}}^{y_m} \phi(t|x) dt + (\lambda - 1)x \int_{\underline{y}}^{y_m} H(t|x) \ln G(t) dt \leq 0$$

which implies  $\lambda > 0$ . Thus,  $E \pi_1(y_m) = 0$ . This violates the assumption.

In summary, there does not exist a reservation price under which the symmetric free-entry equilibrium  $(n, x)$  determined by equations (R&D), (EEa), and (EEb) solves the buyer's optimization problem (17).

Q.E.D.

If the buyer can control each firm's R&D decision or the R&D investment is observable to him, he can force the firms to invest in R&D as described by (17). Proposition 5 says that there does not exist a reservation price under which the symmetric free-entry equilibrium reaches the buyer's optimum (17). In other words, the buyer's ideal optimum in (17) cannot be supported by any symmetric free-entry equilibrium. Therefore, the buyer would not want the firms to make their own R&D and entry decisions.

From (21),  $\frac{\partial \pi}{\partial x} < 0$ , each firm's marginal profit of R&D investment is negative at the buyer's optimum. That is, the buyer would require each active firm to invest more than it wants to. That implies that there will exist a moral hazard problem if the buyer is unable to control the R&D decision  $x$  or if  $x$  is unobservable to the buyer. Thus the buyer has to take each firm's R&D decision as a constraint. He should then solve the optimization problem (17) subject to an additional constraint (R&D). Solving this optimization problem, we know the following: If the opportunity cost  $y_0$  is high, the buyer should choose  $r = \bar{y}$ . If  $y_0$  is relatively low, he should choose  $r$  such that

$$y_0 = r + \left(1 - \frac{\lambda}{n}\right) \frac{H(r|x)}{h(r|x)} - \frac{\mu}{n} \frac{H_x(r|x)}{h(r|x)}$$

where  $n$ ,  $x$ , and the multipliers  $\lambda$  and  $\mu$  are simultaneously determined by the other first-order conditions including (EE) and (R&D). It can also be shown that  $\lambda > 0$  and  $\mu > 0$ . That is, both the nonnegative profits constraint (EEa) and the R&D decision constraint (R&D) are binding. The buyer control free entry by offering a reservation price determined by the above equation. The optimal number of firms is usually more than one under free entry. Because the effect of moral hazard interacts with the effect of asymmetric information, the optimal reservation price  $r$  for the buyer may be either higher than or lower than his opportunity cost  $y_0$ .

### 5. Social Optimality

The expected social costs include the expected production cost, the bid preparation cost, the R&D cost, and the opportunity cost:

$$ESC = \int_{\underline{y}}^{\hat{y}} y d(1 - G(y)^{nx}) + nKH(\hat{y}|x) + nC(x) + y_0 G(\hat{y})^{nx}. \quad (22)$$

Comparing (22) with (10), we know  $ESC = EBC - nE\pi_n$ . That is, the buyer cares about the firms' expected profits which are the transfers from the buyer to the firms, but society does not care. What the society cares about is the total combination of costs on R&D, production, and bid preparation. Under free entry, the expected social cost will be the same as the buyer's expected cost. The social optimization problem is to choose  $(r, n, x, \hat{y})$  to minimize the expected social cost  $ESC(r, n, x, \hat{y})$ .

As discussed in Proposition 4, I first consider the fixed-scale R&D technology. I also assume that there is no bid-preparation cost in this case. Then the expected social costs of procurements can simply be written as

$$ESC(r, n) = \underline{y} + (y_0 - r)G(r)^n + \int_{\underline{y}}^r G(t)^n dt + nC. \quad (23)$$

The social planner wants to choose  $(r, n)$  to minimize the expected social cost function (23) subject to the firm's nonnegative profit constraint (13). I have

**Proposition 6:** Suppose  $K = 0$ ,  $C > 0$ , and  $E\pi_1(y_m) > 0$ , then  $r = y_m$  is socially optimal and each firm earns positive expected profits.

**Proof:** The proof is similar to the proof of Proposition 4. Because of the continuity of  $ESC(r, n)$  and the compactness of the constraints, there exists a solution to the minimization problem (23) with (13). Let  $(r^*, n^*)$  be a solution. Let  $L = -ESC(r, n) + \lambda E\pi_n(r)$ ,  $\phi_\lambda(r, n) = \frac{\partial L}{\partial r}$ , and  $\psi_\lambda(r, n) = \frac{\partial L}{\partial n}$ , where  $\lambda \geq 0$  is the multiplier for the constraint (13). We can calculate

$$\phi_\lambda(r, n) = nG(r)^{n-1}f(r) \left[ y_0 - r + \frac{\lambda}{n} \frac{F(r)}{f(r)} \right]$$

and

$$\psi_\lambda(r, n) = -(y_0 - r)G(r)^n \ln G(r) - C - \int_{\underline{y}}^r G(t)^{n-1} [G(t) - \lambda F(t)] \ln G(t) dt.$$

The first-order conditions give  $\phi_\lambda(r, n) = 0$ ,  $\psi_\lambda(r, n) = 0$ , and  $\lambda E\pi_n(r) = 0$  for interior solution.

First, consider the case  $y_0 \geq \bar{y}$ , then

$$\phi_\lambda(r, n) \geq nG(r)^{n-1}f(r) \left[ \bar{y} - r + \frac{\lambda}{n} \frac{F(r)}{f(r)} \right] > 0$$

for all  $r < \bar{y}$ . Thus  $r^* = \bar{y}$ . Suppose  $n^* = 1$ , then  $\lambda = 0$  because  $\lambda E\pi_1(\bar{y}) = 0$  and  $E\pi_1(\bar{y}) > 0$  by the assumption. Suppose  $n^* > 1$ , then the first-order conditions give  $\psi_\lambda(\bar{y}, n^*) = 0$ . If  $\lambda > 0$ , then  $\psi_\lambda(\bar{y}, n^*) = 0$  implies  $\int_{\underline{y}}^{\bar{y}} G(t)^{n^*} \ln G(t) dt + C < 0$ . But

$$\int_{\underline{y}}^{\bar{y}} G(t)^{n^*-1} [F(t) + G(t) \ln G(t)] dt > 0$$

and hence  $E\pi_{n^*}(\bar{y}) = \int_{\underline{y}}^{\bar{y}} G(t)^{n^*-1} F(t) dt - C > 0$  which implies  $\lambda = 0$ . This contradicts to  $\lambda > 0$ . Thus,  $\lambda = 0$ . Then  $\psi_0(\bar{y}, n^*) = 0$  can be written as  $\int_{\underline{y}}^{\bar{y}} G(t)^{n^*} \ln G(t) dt + C = 0$ . Thus  $E\pi_{n^*}(\bar{y}) > 0$ .

Second, consider the case  $y_0 \geq \bar{y}$ . Suppose  $n^* = 1$ . If  $r^* = \bar{y}$  then  $\phi_\lambda(\bar{y}, 1) = f(\bar{y})(y_0 - \bar{y}) + \lambda/n \geq 0$  which implies  $\lambda > 0$  and hence  $E\pi_1(\bar{y}) = 0$ . But that contradicts to the assumption  $E\pi_1(\bar{y}) > 0$ . This contradiction means  $r^* < \bar{y}$ . Then  $\phi_\lambda(r^*, 1) = 0$ . That is,  $y_0 = r^* - \lambda F(r^*)/f(r^*)$  which implies  $r^* \geq y_0$ . If  $r^* > y_0$  then  $E\pi_1(r^*) > E\pi_1(y_0) > 0$  by the assumption. Then  $\lambda = 0$  which implies  $r^* = y_0$ . This is a contradiction. Thus,  $r^* = y_0$ . Suppose  $n^* > 1$ , then the first-order condition gives  $\psi_\lambda(r^*, n^*) = 0$ . If  $r^* = \bar{y}$ , then  $\psi_\lambda(\bar{y}, n^*) = 0$  becomes

$$C + \int_{\underline{y}}^{\bar{y}} G(t)^{n^*-1} [G(t) - \lambda F(t)] \ln G(t) dt = 0.$$

If  $\lambda > 0$  then  $C + \int_{\underline{y}}^{\bar{y}} G(t)^{n^*} \ln G(t) dt < 0$  which implies  $E\pi_{n^*}(\bar{y}) = \int_{\underline{y}}^{\bar{y}} G(t)^{n^*-1} F(t) dt - C > 0$ . Then  $\lambda = 0$ . This contradicts to  $\lambda > 0$ . Thus,  $\lambda = 0$ . Then  $\phi_0(r, n^*) = n^* G(r)^{n^*-1} f(r)(y_0 - r) < 0$  when  $r$  is less than but very close to  $\bar{y}$ . Thus  $r^*$  cannot be  $\bar{y}$ . In other words,  $r^* < \bar{y}$ . Then  $\phi_\lambda(r^*, n^*) = 0$ . That is,

$$y_0 = r^* - \frac{\lambda}{n^*} \frac{F(r^*)}{f(r^*)}.$$

Now, if  $\lambda > 0$  then  $\psi_\lambda(r^*, n^*) = 0$  implies  $C + \int_{\underline{y}}^{r^*} G(t)^{n^*} \ln G(t) dt < 0$ , which also implies

$$E\pi_{n^*}(r^*) = \int_{\underline{y}}^{r^*} G(t)^{n^*-1} F(t) dt > 0.$$

Then  $\lambda = 0$  which contradicts to  $\lambda > 0$ . Thus,  $\lambda = 0$  which implies  $r^* = y_0$  and  $E\pi_{n^*}(y_0) > 0$ .

In summary, we have shown  $r^* = y_m$  and  $E \pi_{n^*}(y_m) > 0$ .

Q.E.D.

From Proposition 6, the socially optimal reservation price is  $r^* = y_m$  and society allows each firm to earn positive expected profits. The constraint (13) is not binding. The social optimal number of firms  $n^*$  is determined by minimizing  $ESC(y_m, n)$  with respect to  $n$ . Since

$$\frac{\partial ESC(y_m, n)}{\partial n} = C + \int_{\underline{y}}^{y_m} G(t)^n \ln G(t) dt$$

and  $ESC(y_m, n)$  is convex in  $n$ , then  $n^*$  is determined as the following: If  $C + \int_{\underline{y}}^{y_m} G(t) \ln G(t) dt \geq 0$  then  $n^* = 1$ . If  $C + \int_{\underline{y}}^{y_m} G(t) \ln G(t) dt < 0$  then  $n^* > 1$  and satisfies

$$C + \int_{\underline{y}}^{y_m} G(t)^{n^*} \ln G(t) dt = 0.$$

Therefore, society may prefer more than one firm to conduct private R&D under the fixed-scale R&D technology.

The implications of Proposition 4 and 6 are the following: In the first case where the opportunity cost  $y_0$  is high, the buyer should select a socially optimal reservation price  $r = \bar{y}$ , the highest possible cost observation by the firms. In other words, both the buyer and society agree that a reservation price is not necessary. Let  $n_b^*$  be the optimal number of firms for the buyer which is determined by Proposition 4, then  $n_b^* > 1$  and  $E \pi_{n_b^*}(\bar{y}) = 0$ . Notice that  $ESC(\bar{y}, n)$  is strictly convex in  $n$  and

$$ESC(\bar{y}, n) - ESC(\bar{y}, n - 1) = -E \pi_n(\bar{y})$$

for all  $n > 1$ . Then  $ESC(\bar{y}, n_b^*) = ESC(\bar{y}, n_b^* - 1)$ . Thus,  $n_b^* - 1 < n^* < n_b^*$ . Because an integer must be picked, the social optimal number of firms will be the same as the buyer's optimal number of firms. Therefore, free entry and a first-price sealed-bid auction without any reservation price achieve the social optimum. This result was also observed by McAfee and McMillan (1987a).

In the second case where the opportunity cost  $y_0$  is low (lower than the highest possible production cost observation  $\bar{y}$ ), however, the buyer intends to offer a lower reservation price relative to the social optimum, i.e.  $r_b^* < y_0$ . That may cause fewer number of firms to enter the R&D process, relative to the social optimum. In fact, since  $n_b^* > 1$  and  $E \pi_{n_b^*}(y_0) > E \pi_{n_b^*}(r_b^*) = 0$  from Proposition 4 and since

$$ESC(y_0, n) - ESC(y_0, n - 1) = -E \pi_n(y_0)$$

for  $n > 1$ , we have  $ESC(y_0, n_b^*) < ESC(y_0, n_b^* - 1)$ . On the other hand,



$$\frac{\partial ESC(y_0, n)}{\partial n} \Big|_{n=n_b^*-1} = \int_{\underline{y}}^{y_0} G(t)^{n_b^*-1} \left[ F(t) + G(t) \ln G(t) \right] dt - E\pi_{n_b^*}(y_0) < 0.$$

and  $ESC(y_0, n)$  is convex in  $n$ . Thus, we have either  $n_b^* - 1 < n^* < n_b^*$  or  $n^* \geq n_b^*$ . Given the integer problem, the socially optimal number of firms will be at least as larger as the buyer's optimal number of firms. In summary, we have the following Corollary of Proposition 4 and 6:

**Corollary:** Under the fixed-scale R&D technology, if  $E\pi_1(y_m) > 0$ , then the buyer prefers free entry, society does not, and the following hold as well:

- (i) If  $y_0 \geq \bar{y} + 1/f(\bar{y})$  then the buyer's choices of a reservation price and number of firms are socially optimal;
- (ii) If  $y_0 \leq \bar{y}$  then the buyer chooses a reservation price lower than the socially optimal value and a number of firms no larger than the socially optimal number.

When the R&D technology is subject to constant marginal returns to scale on expenditure and there is a bid-preparation cost, the social planner wants to minimize the expected social costs (22) subject to (13), (R&D), and (FE). Then I have

**Proposition 7:** Suppose  $C_1$ ,  $C_2$ , and  $E\pi_1(y_m - K)$  are all positive, then  $n = 1$  with  $r = y_m$  and  $\hat{y} = y_m - K$  is socially optimal.

**Proof:** Because of the continuity of  $ESC(r, n, x, \hat{y})$  and the compactness of the constraints, there exists a solution  $(r^*, n^*, x^*, \hat{y}^*)$  to the social optimization problem. Let  $\lambda \geq 0$  and  $\mu$  be the multipliers for the constraint (13) and (R&D), respectively, and

$$L = -ESC(r, n, x, \hat{y}) + \lambda E\pi_n(r, x) + \mu \left[ - \int_{\underline{y}}^{\hat{y}} G(t)^{nx} \ln G(t) dt - C_1 \right]$$

We can calculate

$$\frac{\partial L}{\partial \hat{y}} = nh(\hat{y} | x) G(\hat{y})^{(n-1)x} \left[ y_0 - \hat{y} + \frac{\lambda}{n} \frac{H(\hat{y} | x)}{h(\hat{y} | x)} + \frac{\mu}{n} \frac{H_x(\hat{y} | x)}{h(\hat{y} | x)} \right] - nKh(\hat{y} | x)$$

$$\begin{aligned} \frac{\partial L}{\partial x} = & -n(y_0 - \hat{y}) G(\hat{y})^{nx} \ln G(\hat{y}) - nKH_x(\hat{y} | x) - n \left[ \int_{\underline{y}}^{\hat{y}} G(t)^{nx} \ln G(t) dt + C_1 \right] \\ & + \lambda(n-1) \int_{\underline{y}}^{\hat{y}} G(t)^{(n-1)x} H(t | x) \ln G(t) dt - n \mu \int_{\underline{y}}^{\hat{y}} G(t)^{nx} \ln^2 G(t) dt \end{aligned}$$

$$\frac{\partial L}{\partial n} = -x(y_0 - \hat{y}) G(\hat{y})^{nx} \ln G(\hat{y}) - KH(\hat{y} | x)$$

$$+ \lambda x \int_{\underline{y}}^{\hat{y}} G(t)^{n-1} x H(t|x) \ln G(t) dt - \mu x \int_{\underline{y}}^{\hat{y}} G(t)^{nx} \ln^2 G(t) dt$$

Since  $C_1 > 0$  and  $C_2 > 0$ , constraint (13) implies  $x = 0$  and  $x = +\infty$  cannot be a solution. Thus, the first-order condition gives  $\frac{\partial L}{\partial x} = 0$ . Then

$$\begin{aligned} n \frac{\partial L}{\partial n} &= n \frac{\partial L}{\partial n} - x \frac{\partial L}{\partial x} \\ &= -nC_2 - nK \left[ H(\hat{y}|x) - x H_x(\hat{y}|x) \right] + \lambda x \int_{\underline{y}}^{\hat{y}} G(t)^{(n-1)x} H(t|x) \ln G(t) dt \\ &< 0. \end{aligned}$$

Therefore,  $n^* = 1$ . Then (FE) implies  $\hat{y} = r - K$  and the first-order conditions  $\frac{\partial L}{\partial \hat{y}} = 0$  and  $\frac{\partial L}{\partial x} = 0$  become

$$y_0 - r + \lambda \frac{H(r-K|x)}{h(r-K|x)} + \mu \frac{H_x(r-K|x)}{h(r-K|x)} = 0 \quad (24)$$

and

$$(y_0 - \hat{y}) G(r-K)^x \ln G(r-K) + \mu \int_{\underline{y}}^{r-K} G(t)^x \ln^2 G(t) dt = 0. \quad (25)$$

Consider the case  $y_0 \geq \bar{y}$ . If  $r^* < \bar{y}$  then  $r^* < y_0$ . Then (24) implies  $\mu < 0$  and (25) implies  $\mu > 0$ . This contradiction means  $r^* = \bar{y}$  and  $\hat{y}^* = \bar{y} - K$ .

Consider the case  $y_0 < \bar{y}$ . If  $r^* < y_0$  then (24) implies  $\mu < 0$  and (25) implies  $\mu > 0$ . This is a contradiction. If  $y_0 < r^* \leq \bar{y}$  then (25) implies  $\mu < 0$  and hence (24) implies  $\lambda > 0$ . Thus,  $E\pi_1(r^* - K) = 0$ . But  $E\pi_1(r^* - K) > E\pi_1(y_0 - K) > 0$  by the assumption. This is also a contradiction. Therefore,  $r^* = y_0$  and  $\hat{y}^* = y_0 - K$ .

Q.E.D.

Even if there are R&D decisions and bid-preparation costs, setting the buyer's reservation price  $r$  equal to the minimum of the opportunity cost  $y_0$  and the highest possible production cost  $\bar{y}$  to be observed by the firms is socially optimal. The most interesting result is that society prefers only one firm to conduct R&D and production when R&D is subject to constant marginal returns to scale on expenditure. Remember that R&D is an independent drawing process and the R&D outcome of  $n$  firms will be the same as the R&D outcome of one firm that invests the same amount as all  $n$  firms. But, because of the fixed cost of R&D, more firms participating in R&D result in higher total R&D

costs. Thus, for society, one firm conducting all of the R&D and production is more efficient than more firms. Contrary to the social optimum, the buyer usually prefers more than one firm to enter both R&D and bidding processes.

From the above comparison analysis, we have seen that the optimal policies from the point of view of the buyer and society are different under two types of R&D technology: the fixed-scale and the variable scale with constant marginal returns. The type of R&D technology plays an important role in determining optimal R&D and procurement policies for the buyer and for society. When R&D is subject to diminishing or increasing marginal returns to scale on expenditure, we expect that some of these results will also change. Further research is needed on this topic.

## 6. Remarks

I have presented a model of private R&D and public procurement with entry. From the above analysis, when R&D technology is subject to constant marginal returns to scale, society prefers to have only one firm conduct all of the R&D and production and the buyer usually prefers more firms to invest in private R&D. But the buyer still has to pay the total R&D costs of all firms even if only one contractor is chosen for production. Therefore, the buyer should like R&D to be conducted efficiently. This raises the question whether there exist alternative and more efficient ways to manage R&D activities. One way might be to have the buyer do the R&D himself and then release the R&D outcomes to potential firms. The buyer could also hire an agency (private or public) to conduct R&D and force the agency to transfer the R&D outcomes to potential producers. That would eliminate the duplication of effort that occurs when several firms conduct R&D at the same time. For instance, in some defense procurement cases, a government agency (e.g. NASA, DOD) conducts the basic research and may also develop the new products. Then, the government agency transfers the technology information to potential contractors for production and chooses the most efficient contractor to produce the product.

There are some disadvantages in releasing or transferring such R&D information. (i) **Credibility:** French and McCormick (1984) have argued that if the buyer does R&D himself, he has an incentive to provide optimistic information about the technology and demand because of the conflicts of interests between him and the firms. Unless this incentive can be controlled, the buyer's information may be ignored by the firms. (ii) **Transferability:** Some technological information (or physical capital, human capital, and so on) obtained by the buyer or the hired agency may not be easily transferable (Williamson 1976, Laffont and Tirole 1988). (iii) **Learning costs:** it takes time or effort for the firms to understand the technological information or prototypes. There are learning costs which will be incurred before production can begin. It would be desirable to investigate and compare different arrangements of R&D management and to identify the advantages and disadvantages of each. Successful modelling will certainly help us better understand current practice in government R&D management.

## Footnotes

1. For a survey on auctions and bidding, see McAfee and McMillan (1987c). For a survey on the economic theory of procurement and contracting, see Besen and Terasawa (1987).
2. McAfee and McMillan (1987b) allow the number of actual bidders to be stochastic, but the probability of any subset of potential bidders becoming the set of actual bidders is assumed to be exogenous and independent of their types. They have shown that the optimal auction is the same whether or not the risk-neutral bidders know who their competitors are.
3. See Riordan and Sappington (1987) and Dasgupta and Spulber (1989) for these results on procurements.
4. Rob (1986) and Tan (1988) have formally incorporated R&D activities into competitive procurement processes for a given number of firms and characterized the equilibrium investment level in R&D and the optimal procurement contract.
5. This R&D process of cost reduction is just an independent one-shot drawing process. It is different from a sequential search process. The first  $x$  dollars have the same effect on the R&D outcome as the last  $x$  dollars. To some extent, this process is subject to constant marginal returns to expenditure in R&D. I consider this special technology to simplify analyzing the free entry equilibrium in this paper. The analysis should be extended to more general cases, such as the diminishing marginal return R&D technologies which are considered in Tan (1988).
6. I consider the first-price sealed-bid auction in this paper because it is often used in practical procurement processes. The second-price sealed-bid auction with the same reservation price will not change the firms' net expected profits from bidding and hence should give the same results. It would be interesting to look at the effects of oral auctions on the firms' R&D investment strategies (possibly asymmetric) by allowing some firms to have information advantages before R&D. I thank Preston McAfee for this interesting point.
7. Under the first-price sealed-bid auction, if  $[1 - F(y)] / f(y)$  is nonincreasing in  $y$  and  $B_i(y) = B_j(y)$  for all  $i, j = 1, \dots, n$ , I can show that there do not exist any asymmetric (bidding and investment) equilibria at both (i) and (ii). Under the second-price sealed-bid auction, the bidding firms bid their true observations of production costs and asymmetric investment equilibria in (ii) always exist. Coordination is needed for equilibrium selection in this case. I ignore such problems in this paper.

8. Since firm  $i$  is unable to observe the other firms' investment levels  $x_{-i}$ , its bidding strategy  $B_i$  can not depend on  $x_{-i}$  directly. Symmetric beliefs of each firm enable us to consider symmetric bidding strategies (see also Footnote 6). I thank Tom Palfrey for this helpful comment.
9. This proof is based on Proposition 2. An alternative proof of Proposition 3 is to construct a compact, convex set  $S$  and a continuous mapping from  $S$  to  $S$ , based upon the equations (EE), (R&D), and (FE), and to use the Brouwer's fixed point theorem. Interested readers can get the manuscript of the second proof from the author.

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